ON THE PROPERTIES OF THE MOTION OF CERTAIN IDEAL GASES FOLLOWING AN EXPLOSION AT A POINT

(O SVOISTVAKH DVIZHENIIA NEKOTORYKH IDEAL'NYKH SRED PRI TOCHECHNOM VZRYVE)

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N.N. KOTCHINA
(Moscow)

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The paper deals with the problem of the explosion at a point in an ideal gas. The properties of the gas are close to one whose isentropic exponent $\gamma = 7$. The unknown functions are written in the form of power series in time, and the terms of the series are determined up to an arbitrary order.

1. Sedov [1] has shown that the problem of a strong explosion in an ideal gas whose isentropic exponent $\gamma = 7$ possesses a simple, exact solution. The solution can be written

$$v = \frac{1}{10}\frac{r}{t}, \quad \rho = \frac{4}{3}\rho_1^{\frac{6}{5}} E^{-\frac{1}{5}} r t^{-\frac{2}{5}}, \quad p = \frac{1}{25}\rho_1^{\frac{6}{5}} E^{-\frac{1}{5}} r^3 t^{-\frac{12}{5}}, \quad E_0 = \frac{2\pi E}{225} \qquad (1.1)$$

Here ρ_1 denotes the density of the gas at rest ahead of the shock front, E denotes a quantity proportional to the total energy E_0 of the explosion, so that the radius of the spherical shock is

$$r_{2a} = \left(\frac{E}{\rho_1}\right)^{\frac{1}{5}} t^{\frac{2}{5}}$$

The corresponding linearized problem (when the linearization is performed around the solution (1.1)) has been studied in [2-4] under a number of additional assumptions.

We shall base our solution on the formulation of the problem given in [4] and will assume that the explosion occurs at a point in a gas whose internal energy is described by the relation

$$\varepsilon(p, \rho) = \frac{p_0}{\rho_0}\left[\frac{P}{6R} + \sum_{k=1}^{\infty} D_k \left(\frac{P}{R^7}\right)^{1-nk}\right] \qquad (1.2)$$
$$\left(R = \frac{\rho}{\rho_0}, \quad P = \frac{p}{p_0}, \quad 0 < n = \frac{1}{l} \leqslant 1\right)$$

Here ρ_0 and p_0 denote certain constants whose dimensions, are those of density and pressure, respectively. The symbols D_k denote arbitrary constants, l is an integer, and the series is convergent. It follows from the argument in [1,4] that the equation of state of the gas under consideration can have a more general form than Clapeyron's equation. Equation (1.2) imposes a limitation only on the isentropic equation which must have the form characteristic of an ideal gas with an exponent $\gamma = 7$. The temperature and entropy have the forms

$$T = \Phi(\Psi)\left[\frac{R^6}{6} + \sum_{k=1}^{\infty} D_k(1-nk)\Psi^{-nk}\right], \qquad S = S_0 + \frac{p_0}{\rho_0}\int\frac{d\Psi}{\Phi(\Psi)} \qquad \left(\Psi = \frac{P}{R^7}\right)$$

and $\Phi(\Psi)$ is an arbitrary function.

We shall seek a solution of the above problem in the form [2,4]

$$u = \frac{1}{10}\frac{r}{t}F(\lambda, \tau), \quad \rho = \frac{4}{3}\rho_1^{\frac{6}{5}}E^{-\frac{1}{5}}rt^{-\frac{2}{5}}G(\lambda, \tau), \quad p = \frac{1}{25}\rho_1^{\frac{6}{5}}E^{-\frac{1}{5}}rt^{-\frac{12}{5}}H(\lambda, \tau) \tag{1.3}$$

$$\lambda = \rho_1^{\frac{1}{5}}E^{-\frac{1}{5}}rt^{-\frac{2}{5}}, \qquad \tau = (p_0E^{-\frac{2}{5}}\rho_0^{-\frac{3}{5}})^n t^{\frac{6}{5}n} \tag{1.4}$$

Assuming that the functions $F(\lambda, \tau)$, $G(\lambda, \tau)$, and $H(\lambda, \tau)$ can be expanded in series in powers of τ, we write

$$F(\lambda, \tau) = 1 + \sum_{k=1}^{\infty} f_k(\lambda)\tau^k, \qquad G(\lambda, \tau) = 1 + \sum_{k=1}^{\infty} g_k(\lambda)\tau^k$$
$$H(\lambda, \tau) = 1 + \sum_{k=1}^{\infty} h_k(\lambda)\tau_k \tag{1.5}$$

Substituting (1.3)-(1.5) into the equation of motion, we obtain the following nonhomogeneous system of linear differential equations (Euler's equations) for the functions $f_k(\lambda)$, $g_k(\lambda)$, and $h_k(\lambda)$:

$$\begin{aligned} -3\lambda f_k' + 3\lambda h_k' + (12nk-8)f_k - 9g_k + 9h_k + F_k^{(1)} &= 0 \\ \lambda f_k' - 3\lambda g_k' + 4f_k + 12nkg_k + F_k^{(2)} &= 0 \\ 21\lambda g_k' - 3\lambda h_k' - 4f_k - 84nkg_k + 12nkh_k + F_k^{(3)} &= 0 \end{aligned} \tag{1.6}$$

where

$$F_k^{(1)} = \sum_{i=1}^{k-1}\left\{[(12ni-8)f_i - 3\lambda f_i']g_{k-i} + (f_i + \lambda f_i')\left[f_{k-i} + \sum_{j=1}^{k-i-1} f_j g_{k-i-j}\right]\right\}$$

$$F_k^{(2)} = \sum_{i=1}^{k-1}\{(4f_i + \lambda f_i')g_{k-i} + \lambda f_i g'_{k-i}\} \tag{1.7}$$

$$F_k^{(3)} = \sum_{i=1}^{k-1} \Big\{3h_i\,[7\lambda g'_{k-i} + 4n\,(8i - 7k)\,g_{k-i}] - 3\lambda h_i' g_{k-i} + f_{k-i}\,[\lambda h_i' - 4h_i - - 7\lambda g'_i - 4g_i] + f_i \sum_{j=1}^{k-i-1} [g_j\,(\lambda h'_{k-i-j} - 4h_{k-i-j}) - 7\lambda g_j' h_{k-i-j}]\Big\}$$

It is easy to verify that the right-hand sides of Equations (1.6), $F_k^{(i)}(\lambda)$ are of the form

$$F_k^{(i)}\lambda = -\sum_{\mu=4}^{\omega_k} \beta_{\mu k}^{(i)} \lambda^{\gamma_{\mu k}} \qquad (\beta_{\mu k}^{(i)},\ \gamma_{\mu k} = \text{const}) \tag{1.8}$$

Assuming that the constants $B_{\mu k}$ and $\gamma_{\mu k}$ are known, and making use of Equations (1.8), we can find the solution of the system of Equations (1.6)

$$\begin{aligned} f_k(\lambda) &= -3 \sum_{i=2}^{3} C_k^{(i)} \lambda^{\alpha_i(nk)} + \sum_{\mu=4}^{\omega_k} \theta_{\mu k}^{(1)} \lambda^{\gamma_{\mu k}} = \sum_{\mu=1}^{\omega_k} \theta_{\mu k}^{(1)} \lambda^{\gamma_{\mu k}} \\ g_k(\lambda) &= \sum_{i=1}^{3} \mu_k^{(i)} C_k^{(i)} \lambda^{\alpha_i(nk)} + \sum_{\mu=4}^{\omega_k} \theta_{\mu k}^{(2)} \lambda^{\gamma_{\mu k}} = \sum_{\mu=1}^{\omega_k} \theta_{\mu k}^{(2)} \lambda^{\gamma_{\mu k}} \\ h_k(\lambda) &= \sum_{i=1}^{3} \sigma_k^{(i)} C_k^{(i)} \lambda^{\alpha_i(nk)} + \sum_{\mu=4}^{\omega_k} \theta_{\mu k}^{(3)} \lambda^{\gamma_{\mu k}} = \sum_{\mu=1}^{\omega_k} \theta_{\mu k}^{(3)} \lambda^{\gamma_{\mu k}} \end{aligned} \tag{1.9}$$

Here $C_k^{(i)}$ are constants of integration

$$\alpha_1(x) = 4x, \qquad \alpha_{2,3}(x) = \frac{-(12x + 17) \pm \sqrt{7\,(48x^2 + 40x + 7)}}{4}$$

$$\mu_k^{(1)} = 1, \qquad \sigma_k^{(1)} = \frac{3}{\alpha_1(nk) + 3}, \qquad \mu_k^{(i)} = \frac{4 + \alpha_i(nk)}{4nk - \alpha_i(nk)}$$

$$\sigma_k^{(i)} = \frac{24 + 7\alpha_i(nk)}{4nk - \alpha_i(nk)}\ (i = 2,\ 3), \qquad \theta_{\mu k}^{(i)} = \frac{1}{D_{\mu k}} \Big(\sum_{j=1}^{3} \beta_{\mu k}^{(j)} D_{\mu k}^{(ji)}\Big) \quad (\mu \geqslant 4)$$

$$D_{\mu k} = 9 \begin{vmatrix} -3\gamma_{\mu k} + 12nk - 8 & -3 & \gamma_{\mu k} + 3 \\ \gamma_{\mu k} + 4 & -\gamma_{\mu k} + 4nk & 0 \\ -4 & 7\,(\gamma_{\mu k} - 4nk) & -\gamma_{\mu k} + 4nk \end{vmatrix} \tag{1.10}$$

$D_{\mu k}^{(ij)}$ are the adjoints of the determinant $D_{\mu k}$.

The relations in (1.7) and (1.8) show that the solution of the problem of a point-explosion by the method of successive approximations reduces itself to the determination of the constants $B_{\mu k}{}^{(i)}$ and $\gamma_{\mu k}$. Hence, the constants $C_k{}^{(i)}$ can be determined for the boundary conditions which correspond to the numerous individual problems mentioned before [2-4].

2. We note that for the first approximation ($k = 1$), Equations (1.6) represent a homogeneous system, so that in Equations (1.8) it is necessary to put [2,4]

$$\beta_{\mu 1}{}^{(i)} \equiv 0 \qquad (\mu = 1,\ 2,\ 3) \tag{2.1}$$

It is easy to verify that the functions $F_k{}^{(l)}(\lambda)$, $f_k(\lambda)$, $g_k(\lambda)$ and $h_k(\lambda)$ can be assumed to be of the form

$$\begin{aligned} F_k{}^{(l)}(\lambda) &= \sum_{p=1}^{k-1} F_{kp}{}^{(l)}(\lambda), \qquad & f_k(\lambda) &= \sum_{p=0}^{k-1} f_{kp}(\lambda) \\ g_k(\lambda) &= \sum_{p=0}^{k-1} g_{kp}(\lambda), \qquad & h_k(\lambda) &= \sum_{p=0}^{k-1} h_{kp}(\lambda) \end{aligned} \tag{2.2}$$

The functions $f_{k0}(\lambda)$, $g_{k0}(\lambda)$, $h_{k0}(\lambda)$ and $F_{kl}{}^{(l)}(\lambda)$ can be written down at once for arbitrary values of k from Equations (1.7), assuming that in the relations (1.9) all $\theta_{\mu k}{}^{(i)} \equiv 0$ ($\mu \geqslant 4$), i.e. taking into account only the solution of the homogeneous system (1.6). Hence, making use of Equations (1.8)-(1.10), where μ ranges over the interval from 1 to ω_{k1}, we write down the expressions $f_{k1}(\lambda)$, $g_{k1}(\lambda)$, $h_{k1}(\lambda)$, and, making use of these functions, we write down $F_{k2}{}^{(l)}$ etc. It is seen from the formulas in (1.7) that this process will come to an end with $p = k - 1$.

We shall assume that the following relations are valid:

$$f_{ip} = \sum_{\mu} \theta_{\mu i}{}^{(1)} \lambda^{\gamma_{\mu i}}, \qquad g_{ip} = \sum_{\mu} \theta_{\mu i}{}^{(2)} \lambda^{\gamma_{\mu i}}, \qquad h_{ip} = \sum_{\mu} \theta_{\mu i}{}^{(3)} \lambda^{\gamma_{\mu i}} \tag{2.3}$$

Here, the summation extends over $\mu = 1 + \omega_p{}^{(l)}, \ldots, \omega_{p+1}{}^{(i)}$, and ($i \ll k - 1$) $\omega_0{}^{(i)} = 0$, $\omega_1{}^{(i)} = 3$; the quantities $\gamma_{\mu i}$, $\theta_{\mu i}{}^{(l)}$ are known.

Substituting equations analogous to (2.3) into the relations for the functions $F_{kp}{}^{(l)}(\lambda)$, we shall obtain for every function $F_{kp}{}^{(l)}$ a closed expression in terms of known quantities.

We now enumerate the indices and the corresponding coefficients which enter into the expressions for the functions $F_{kp}{}^{(l)}$.

It is possible to verify that the functions $F_{kp}{}^{(l)}$ are given by

$$F_{kp}^{(l)} = \sum_{\mu} \beta_{\mu k}{}^{(l)} \lambda^{\gamma_{\mu k}} \qquad (l = 1,\ 2,\ 3;\ p = 1, \ldots, k-1) \tag{2.4}$$

Here the summation extends over $\mu = 1 + \omega_p^{(k)}, \ldots, \omega_{p+1}^{(k)}$, and the numbers $\omega_{p+1}^{(k)} - \omega_p^{(k)}$ as well as the constants $\gamma_{\mu k}$, $B_{\mu k}^{(l)}$ for these values of μ will be determined.

Making use of Equations (1.8)-(1.10), we write the corresponding expressions for the functions $f_{kp}(\lambda)$, $g_{kp}(\lambda)$, $h_{kp}(\lambda)$

$$f_{kp} = \sum_\mu \theta_{\mu k}{}^{(1)} \lambda^{\gamma_{\mu k}}, \qquad g_{kp} = \sum_\mu \theta_{\mu k}{}^{(2)} \lambda^{\gamma_{\mu k}}, \qquad h_{kp} = \sum_\mu \theta_{\mu k}{}^{(3)} \lambda^{\gamma_{\mu k}} \tag{2.5}$$

Here the summation over μ is analogous to (2.4).

In this manner it is possible to determine (at once for all k's) the corresponding parts of the solution consecutively for $p = 0, 1, \ldots, k - 1$. Hence, for every p, we have found the solution $f_p(\lambda)$, $g_p(\lambda)$, $h_p(\lambda)$.

We note that the same method can be used to write down the solution which corresponds to a more general form for internal energy ($\Delta_k(R)$ denote arbitrary functions).

$$\varepsilon(p, \rho) = \frac{p_0}{\rho_0}\left[\frac{P}{6R} + \sum_{k=1}^{\infty} P^{1-nk} \Delta_k(R)\right] \tag{2.6}$$

We shall refrain from writing down the corresponding expressions for temperature and entropy owing to the complexity of their expressions; for the case $\Delta_k = 0$ $(k > 1)$ in Equation (2.6), the corresponding expressions can be found in [4].

3. We now determine the constants $C_k^{(i)}$. In order to satisfy the condition at the center of symmetry (particle velocity equal to zero) [2, 4], it is necessary to put $C_k^{(3)} \equiv 0$. The expression for the dimensionless radius of the shock wave in terms of time can be assumed to be in the form of a power series in τ

$$\left(\frac{\rho_1}{E}\right)^{\frac{1}{5}} r_2 t^{-\frac{2}{5}} = 1 + \sum_{k=1}^{\infty} A_k \tau^k \tag{3.1}$$

We can determine the constants A_k, $C_k^{(1)}$ and $C_k^{(2)}$ from the shock conditions

$$v_2 = c\left(1 - \frac{\rho_1}{\rho_2}\right), \qquad p_2 = p_1 + \rho_1 c^2\left(1 - \frac{\rho_1}{\rho_2}\right)$$

$$\frac{1}{2} c^2 \left(1 - \frac{\rho_1^2}{\rho_2^2}\right) + \frac{7}{6}\left[\frac{p_1}{\rho_1} - \frac{p_1}{\rho_2} - \frac{\rho_1}{\rho_2}\left(1 - \frac{\rho_1}{\rho_2}\right) c^2\right] + \tag{3.2}$$

$$+ \frac{p_0}{\rho_0} \sum_{k=1}^{\infty} D_k \left\{\left(\frac{P_1}{R_1^7}\right)^{1-nk} - \left\{\frac{1}{R_2^7}\left[P_1 + \frac{\rho_1}{p_0}\left(1 - \frac{\rho_1}{\rho_2}\right) c^2\right]\right\}^{1-nk}\right\} = 0$$

It turns out that these constants are determined by a system of linear equations, the right-hand sides of which contain the parameters of the problem as well as the constants A_q, $C_q^{(1)}$, $C_q^{(2)}$, where $q < k$, which have already been determined from the solution of the preceding system.

4. We now determine the behavior of the solution (1.9) in the neighborhood of the center of symmetry. Assuming that in a certain interval $0 \leqslant R \leqslant R_*$ we have

$$\Delta_k(R) = D_k R^{-\gamma(1-nk)}$$

and retaining in the expressions (1.9) only the leading terms $\theta^{(i)}_{\omega_k k}\lambda^{\gamma_{\omega_k k}}$ $(i = 1, 2, 3)$, we can write down the following solution of (1.5):

$$F(\lambda, \tau) = 1 + \sum_{k=1}^{\infty} \theta^{(1)}_{\omega_k k} x^k, \quad G(\lambda, \tau) = 1 + \sum_{k=1}^{\infty} \theta^{(2)}_{\omega_k k} x^k, \ H(\lambda, \tau) = 1 + \sum_{k=1}^{\infty} \theta^{(3)}_{\omega_k k} x^k \tag{4.1}$$

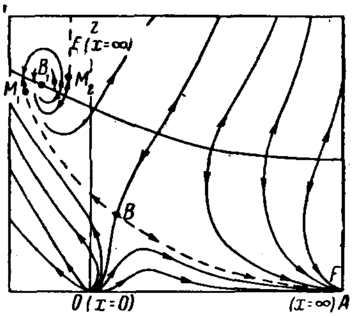
Here

$$x = \tau\lambda^{\alpha_2(n)} = \tau\lambda^{\frac{1}{k}\gamma_{\omega_k k}} \tag{4.2}$$

and the constants $\beta^{(l)}_{\omega_k k}$ have the form $(a_2 = a_2(n))$

$$\begin{aligned}
\beta^{(1)}_{\omega_k k} &= \sum_{i=1}^{k-1} \theta^{(1)}_{\omega_i i} \Big\{ [12ni - 3i\alpha_2 - 8]\, \theta^{(2)}_{\omega_{k-i} k-i} + \\
&\qquad + (1 + i\alpha_2) \Big[\theta^{(1)}_{\omega k-i k-i} + \sum_{j=1}^{k-i-1} \theta^{(1)}_{\omega_j j} \theta^{(2)}_{\omega_{k-i-j} k-i-j} \Big] \Big\} \\
\beta^{(2)}_{\omega_k k} &= (4 + k\alpha_2) \sum_{i=1}^{k-1} \theta^{(1)}_{\omega_i i} \theta^{(2)}_{\omega_{k-i} k-i} \\
\beta^{(3)}_{\omega_k k} &= \sum_{i=1}^{k-1} \Big\{ 3\theta^{(3)}_{\omega_i i} \theta^{(2)}_{\omega_{k-i} k-i} [(7k - 8i)\,\alpha_2 + 4n\,(8i - 7k)] + \\
&\qquad + \theta^{(1)}_{\omega_{k-i} k-i} [(i\alpha_2 - 4)\, \theta^{(3)}_{\omega_i i} - (7i\alpha_2 + 4)\, \theta^{(2)}_{\omega_i i}] + \\
&\qquad + \theta^{(1)}_{\omega_i i} \sum_{j=1}^{k-i-1} \theta^{(2)}_{\omega_j j} \theta^{(3)}_{\omega_{k-i-j} k-i-j} [(k - i - 8j)\,\alpha_2 - 4] \Big\}
\end{aligned} \tag{4.3}$$

The Formulas (1.3), (1.4), (4.1)-(4.3) determine the asymptotic behavior of the solution. This solution is identical with the solution given in [2,4] for the form applicable near the center of symmetry and based on the linearized formulation of the problem of a point-explosion. This shows that the reasoning in [4] concerning the behavior of the characteristics of the motion near the center of symmetry in terms of the magnitude of the parameter n and of the sign of the constant $C_1^{(2)}$ is correct. Furthermore, the conclusion concerning the emergence of a second shock wave for $D_k = 0$ ($k = 0, 1, 2$) for a series of values of n and $C_n^{(2)}$ and its propagation from the center of symmetry behind the first shock is also correct.



The diagram represents the field of integral curves for the equation

$$\frac{dz}{dF} = \frac{2z\,\{7\,(12n - 4\alpha_2 + \alpha_2 F)\,[3\alpha_2 F\,(F - 10) - 10\,(F - 1)\,(12n - 4\alpha_2 + \alpha_2 F)]\}}{(12n - 4\alpha_2 + \alpha_2 F)\,\{-7F\,(F - 10)\,(12n - 4\alpha_2 + \alpha_2 F) + 9\,[7\alpha_2 F - 4\,(\alpha_2 + 3n)]\,z\}} +$$
$$+ \frac{6\alpha_2 z^2\,[9\,(12n - 4\alpha_2 + \alpha_2 F) - 2\alpha_2\,(F - 1)]}{(12n - 4\alpha_2 + \alpha_2 F)\,\{-7F\,(F - 10)\,(12n - 4\alpha_2 + \alpha_2 F) + 9\,[7\alpha_2 F - 4\,(\alpha_2 + 3n)]\,z\}} \quad (4.4)$$

($z = 7\,G/H$). The solution near the center of explosion [2, 4] for the case $0.66978 < n \leqslant 1$ can be reduced to the qualitative investigation and to the integration of the preceding equation.

When $C_1^{(2)} < 0$, the solution of the problem under consideration is continuous, the integral curve which gives the solution of the problem emerges from point $B(F = 1,\ z = 7,\ x = 0)$ which corresponds to the appropriate self-similar case, and enters point $A(F = 10,\ z = 0,\ x = \infty)$ which corresponds to the center of symmetry. When $C_1^{(2)} > 0$, the integral curve which leaves point B reaches point B_1 which turns out to be a focal point. Point B_1 lies on the parabola

$$z = \frac{1}{3\alpha_2^2}\,(12n - 4\alpha_2 + \alpha_2 F)^2 \quad (4.5)$$

on which the parameter x reaches an extremal value. Consequently, on approaching point B_1, the integral curve intersects the parabola (4.5) in an infinite number of points and a continuous solution of the problem proves to be impossible. If in Equation (2.6), in the neighborhood of $R = 0$, it is possible to put $\Delta_k(R) \equiv 0$ for all k's, then a solution with a shock wave can be obtained (the state in the gas ahead of the shock wave is described in the diagram by point M_1 which lies on the dotted curve entering point B_1; the state behind the shock wave is represented by point M_2 which lies on the curve entering point E for which $z = \infty$ and $x = \infty$).

In the interval $0 < n \leqslant 1$ under consideration it is possible to obtain analogous results for other values of n. The second shock wave is obtained as a consequence of the fact that the integral curve of interest to us passes through a singular point which lies on the parabola, (4.5), and that the latter is a focal point.

5. Sedov demonstrated that the solution of any problem of self-similar motion of a perfect gas can be reduced to the qualitative investigation of an ordinary differential Equation (2.1), [1], p. 171 and to the determination of the relevant integral curve; when the values of ν and γ are fixed, the equation depends on two parameters δ and κ.

There exist examples of self-similar problems, and of problems which can be reduced to self-similar problems in which a second shock wave does appear.

The coordinates V of the singular points of Equation (2.1) [1], which are obtained at the points of intersection of the curve along which the integral curves are horizontal with the curve along which the integral curves are vertical, satisfy a given cubic equation. It is possible to show that the latter can be decomposed into a quadratic and a linear equation

$$(\nu - 1) V^2 + [1 - \varkappa - \nu\delta] V + \varkappa\delta = 0, \quad [2 + \nu(\gamma - 1)] V - 2 = 0 \tag{5.1}$$

In the case of two-dimensional symmetry, the foregoing cubic equation, as seen from Equations (5.1), reduces to a quadratic.

We write down the coordinates of the respective points

$$V_1 = \frac{2}{2 + \nu[\gamma - 1]}, \quad z_1 = \frac{2\nu(\gamma - 1)\{2 - [2 + \nu(\gamma - 1)]\delta\}}{\{-2\nu + \varkappa[2 + \nu(\gamma - 1)]\}[2 + \nu(\gamma - 1)]^2} \tag{5.2}$$

$$V_{2,3} = \frac{1}{2(\nu - 1)}[\varkappa + \nu\delta - 1 \mp \sqrt{\varkappa^2 + 2[(2 - \nu)\delta - 1]\varkappa + (\nu\delta - 1)^2}]$$

$$z_{2,3} = [\{\varkappa^2 + 2[(2 - \nu)\delta - 1]\varkappa + [\nu^2 - 2\nu + 2]\delta^2 - 2\delta + 1\} \mp \tag{5.3}$$

$$\mp [\varkappa + (2 - \nu)\delta - 1]\sqrt{\varkappa^2 + 2[(2 - \nu)\delta - 1]\varkappa + (\nu\delta - 1)^2}]\frac{1}{2(\nu - 1)^2} \quad (\nu = 2,3)$$

$$V_2 = \frac{\varkappa\delta}{\varkappa + \delta - 1}, \qquad z_2 = \frac{\delta^2(\delta - 1)^2}{(\varkappa + \delta - 1)^2} \ (\nu = 1)$$

When $\nu = 2$ or $\nu = 3$, the singular points (V_2, z_2) and (V_3, z_3) are real if the parameters δ and κ satisfy the inequality

$$\varkappa^2 + 2[(2-\nu)\delta - 1]\varkappa + (\nu\delta - 1)^2 \geqslant 0$$

If $\delta \geqslant 1$, the inequality is always satisfied; if $\delta < 1$, it is satisfied when

$$\varkappa \leqslant 1 - (2-\nu)\delta - 2\sqrt{(\nu-1)\delta(1-\delta)}$$

and when

$$\varkappa \geqslant 1 - (2-\nu)\delta + 2\sqrt{(\nu-1)\delta(1-\delta)}$$

It is easy to verify directly that the singular points $(V_2,\ z_2)$ and $(V_3,\ z_3)$ lie on the parabola $z = (V = \delta)^2$, on which the parameter $\lambda = r/bt^\delta$ attains an extremal value.

The singular point $(V_1,\ z_1)$ lies above the parabola $z = (V - \delta)^2$ when the following conditions are satisfied

$$1)\ \delta > \frac{2}{2+\nu(\gamma-1)}, \qquad \frac{2\nu\{\gamma+1-\delta[2+\nu(\gamma-1)]\}}{[2+\nu(\gamma-1)]\{2-\delta[2+\nu(\gamma-1)]\}} < \varkappa < \frac{2\nu}{2+\nu(\gamma-1)}$$

$$2)\ \delta < \frac{2}{2+\nu(\gamma-1)}, \qquad \frac{2\nu}{2+\nu(\gamma-1)} < \varkappa < \frac{2\nu\{\gamma+1-\delta[2+\nu(\gamma-1)]\}}{[2+\nu(\gamma-1)]\{2-\delta[2+\nu(\gamma-1)]\}}$$

We shall examine the first case in more detail. It is possible to show that the following inequalities are satisfied: $V_2 < V_3 < V_1$; in addition, if $\delta < 1$, $\kappa > 0$, all integral curves, with the exception of one, have at point $(V_2,\ z_2)$ a smaller, and at point $(V_3,\ z_3)$ a larger slope with respect to the V-axis than the parabola $z = (V - \delta)^2$ does at the same point. Consequently, the direction of motion along the integral curves towards increasing λ's is such that if at the first instant of time, the velocity, the density and the pressure are described by certain step-functions, then, when a cylindrical or spherical piston moves outwards $(r_* = \lambda_*\ bt^\delta)$, there will appear a second shock wave in a certain interval of λ_*, and the second shock wave will follow the first one.

Equation (4.4) represents a special case of Equation (2.1) in [1], when

$$\nu = 3, \qquad \gamma = 7, \qquad \delta = \frac{2}{5}\left[1 - \frac{3n}{\alpha_2(n)}\right], \qquad \varkappa = \frac{6}{35}\left[1 + \frac{3n}{\alpha_2(n)}\right]$$

The special result of the problem of a point-explosion considered in Sections 1-4 consists in the fact that a second shock wave has been found in an example when the motion is not self-similar; it is self-similar only in the neighborhood of the center of symmetry.

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